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Measurement Interpolation Methods for Dual One-Way Ranging Systems

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A dual one-way ranging method is designed to minimize the oscillator noise effect in intersatellite microwave ranging systems by combining two one-way phase measurements. This system requires the measurement timing synchronization of the two measurements. The interpolation of the two measurements toward a common epoch resolves this requirement but causes interpolation error. To reduce the interpolation error, a new method is proposed, which modifies the interpolation sequence. Simulation data based on the Gravity Recovery and Climate Experiment mission are tested with the new method, and it demonstrates the advantage over a conventional interpolation method. Experiments with the flight data agree with the simulation results. In addition to the interpolation error reduction, this method partly evaluates the interpolation error level in the flight data processing.

Nomenclature

$E_{i/j}$ = phase error at the ith satellite, cycle
R = dual one-way range, m
t = nominal observation time, s
Δt_i = timing offset of the ith satellite, s
λ = signal wavelength, m
τ = signal time of flight, s
φ_i = reference/transmitted phase at the ith satellite, cycle
φ_i^j = phases measurements at the ith satellite, cycle
φ^j = received phase from the jth satellite, cycle

I. Introduction

ONE of the ways of measuring intersatellite distance is the use of a microwave ranging system to transmit and receive carrier phase signals, and its measurement accuracy is mainly limited by the instability of the oscillator that drives the phase signals. A dual one-way ranging (DOWR) system minimizes the oscillator noise effect by combining the one-way range measurements from two microwave ranging systems [1,2]. With identical transmission and reception subsystems, each satellite transmits a carrier phase signal to the other satellite. The received signal at each satellite is recorded and later transmitted to a control segment. The frequency fluctuations due to oscillator instability have nearly equal and opposite effects on each satellite's measurement, and summation of these two phases cancels most of the oscillator noise. The combined phase measurement is converted to the biased range between the two satellites with very high precision.

The DOWR system was first implemented on the Gravity Recovery and Climate Experiment (GRACE) mission, a dedicated spaceborne mission designed to map the Earth's gravity field with high accuracy. The mission consists of two coorbiting low Earth orbit satellites separated in orbit by about 220 km [3]. Its DOWR system measures the intersatellite distance with micrometer-level accuracy. Two GRACE satellites have been successfully working since their launch in 2002. The mission lifetime was extended from its design lifetime of five years.

A key requirement for the DOWR is the measurement time synchronization; the measurement epoch of the two satellites' phases should be very close to maximize the noise cancellation. Because the phase noise cancellation is based on the characteristic that the same noise exists in both measurements, time mismatching of the two measurements degrades the noise cancellation performance. The measurement time synchronization process involves the numerical interpolation of each satellite's phase measurement toward a common epoch. If the timing offset, difference between the common epoch, and the actual measurement time is not small, then it may cause numerical interpolation error. This research proposes a new approach to mitigate the interpolation error by a simple modification of the interpolation procedure. The feasibility of the new approach is demonstrated with DOWR simulation results first and then with GRACE flight data.

II. Dual One-Way Ranging and Measurement Interpolation

A. Dual One-Way Ranging Measurements

Two satellites transmit and receive carrier phase signals to and from each other. Phase measurement received at the two satellites at a specified nominal time t can be modeled as follows [4]:

$$\text{Satellite-}A\text{: } \varphi_A^B(t+\Delta t_A) = \varphi_A(t+\Delta t_A) - \varphi^B(t+\Delta t_A) = R(t)/\lambda_{A/B} + E_A \tag{1}$$

$$\text{Satellite-}B\text{: } \varphi_B^A(t+\Delta t_B) = \varphi_B(t+\Delta t_B) - \varphi^A(t+\Delta t_B) = R(t)/\lambda_{B/A} + E_B \tag{2}$$

A satellite's actual measurement time is different from the nominal epoch t by as much as Δt_i. The two phases of Eqs. (1) and (2) are nearly equal in magnitude but opposite in sign after replacing the received phase with the transmitted phase before the signal time of flight, that is, $\varphi^B(t+\Delta t_A) = \varphi_B(t+\Delta t_A - \tau_A^B)$. This opposite signal structure is also applied for the phase noise.

Dual one-way phase is defined as the summation of the two one-way phases of Eqs. (1) and (2). With this summation, most of the phase errors are canceled out except short period variation having a duration less than the signal time of flight. Dual one-way range is

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obtained by multiplying the dual one-way phase by two times the wavelength. Because the two satellites' transmit frequencies are slightly different, the sum of the two wavelengths should be used for the multiplication. Detailed derivations can be found in the literature [4].

B. Measurement Interpolation for Time Synchronization

One of the error sources of the DOWR system is the measurement timing offset Δt_A and Δt_B. The range error due to the timing offsets is proportional to the relative timing offset between the two satellites and the time-of-flight variation during the offset. Detailed derivations of the error can be found in literature [4]. For real-time processing of the DOWR data, the timing offset should be minimized by steering the onboard clock toward a reference time. For post-processing, more accurate timing offset can be obtained by post-processing onboard GPS (Global Positioning System) measurements on the ground. Using the accurate timing offset, the raw one-way phase measurements can be interpolated toward the nominal epoch t of Eq. (1) and then combined. This interpolation process may cause an interpolation error, and reduction of the interpolation error is the objective of this research.

One of the methods used to synchronize the measurement time is interpolating each phase measurement toward a common epoch (nominal time, $T0$) using the measurement time correction estimates. Then combination of the two measurement time corrected phases yields the dual one-way phase at the nominal time. Figure 1 illustrates the interpolation-combination procedure, and it is called "method-1" hereafter. This method is implemented in current GRACE data processing.

The one-way phase contains more noise than the combined phase (dual one-way phase) because the phase noise is not cancelled out yet. To reduce the interpolation error, it is better to interpolate the combined phase instead of the one-way phases. A new method, called "method-2" is proposed to reduce the interpolation error. This method performs two-step measurement interpolations. First, one satellite's phase measurement at $T2$ is interpolated toward the other satellite's measurement time ($T1$), and then the two phase measurements are combined to yield a dual one-way phase at $T1$. The next step is to interpolate the combined phase toward a common epoch ($T0$). Figure 2 illustrates the proposed interpolation method. The question is how to select a phase measurement for the first interpolation. Simulation analysis in the following section will determine this selection process.

C. GRACE DOWR System

The GRACE DOWR system uses K (24 GHz) and Ka (32 GHz) band frequencies, and it is called a K-band ranging (KBR) system.

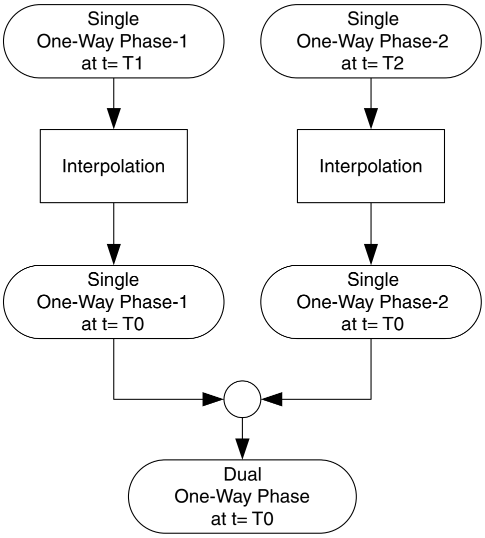


Fig. 1 Phase measurement interpolation, method 1.

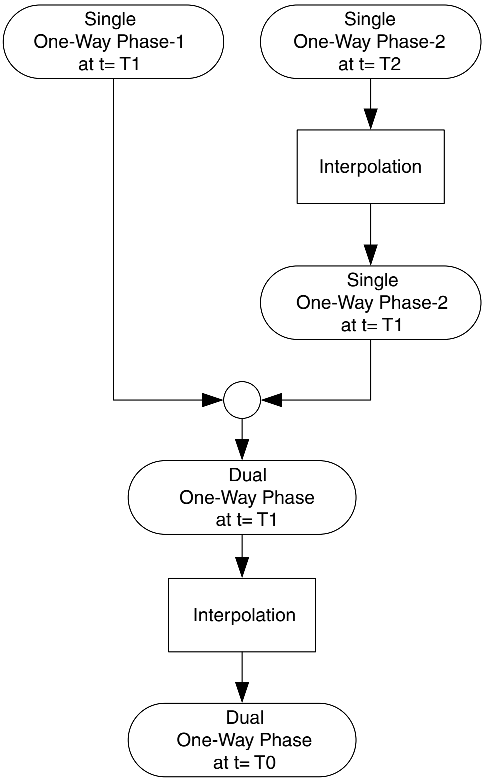


Fig. 2 Phase measurement interpolation, method 2.

Although it is not possible to evaluate the DOWR flight performance directly, certain analyses shows that the accuracy is close to the 10 μm level [5,6], the level predicted from ground tests and numerical simulations [7,8]. The GRACE onboard controller is driven by an ultrastable quartz crystal-type oscillator (USO), and its stability is very high when compared with low-cost quartz crystal-type oscillators. However, the onboard controller counts the time assuming the USO base frequency is exactly same as the design frequency although the actual manufactured USO is slightly different from the design value due to manufacturing limitation. For this reason the GRACE clock always deviates from the reference (GPS) time, and the onboard controller frequently resets its time by comparing it with the GPS time from its onboard GPS receiver. This clock deviation is not any manufacturing problem, but it requires the measurement interpolation for the time synchronization. The sampling interval of the one-way phase measurement is 0.1 s. Because two satellites' carrier frequencies are offset by 0.5 MHz, the received phases, the time integration of the frequency, show a strong linear trend.

The measurement time synchronization requirement for the GRACE is less than 150 ps, equivalent to a 1 μm range error. The GRACE onboard GPS receiver provides this time-tag information, but this level of accuracy is hard to achieve in space in real time. Therefore, ground postprocessing with external data, for example, International GPS Service (IGS) data, is necessary to achieve this level of accuracy [5,6].

A comprehensive GRACE DOWR numerical simulator was developed by the authors during the GRACE development period for prelaunch performance analysis and mission design purposes. The first step of the simulator is simulating satellite orbits under comprehensive dynamics models, for example, gravity field, atmospheric drag, etc. The signal time of flights are computed from those simulated orbits. The phase noise is generated with oscillator design specifications and the Gauss Markov process. From the same phase noise time series, the clock error time series are generated, because the GRACE oscillator drives the DOWR and onboard clock simultaneously. With the clock error and time-of-flight information, the one-way phase measurements at the nominal time plus timing offset are generated. Other error sources applied to the one-way phases include multipath, ionosphere, system noise, amplitude/phase modulation conversion error, and attitude determination error. Details on the simulation procedure can be found in the literature [7,8]. The simulation data were validated by comparing with the GRACE flight data, and the power spectral density level agreed with the flight data.

III. Simulation Data Analysis

To test the proposed interpolation method, GRACE DOWR simulation data are applied. The test results may depend on the integrity of the simulation data, for example, noise level and characteristics, although the simulation data were validated with the flight data. To support the results, an additional test is performed with a simple linear curve because it provides an analytic truth value for comparison and does not depend on the simulation data integrity. When interpolation method 2 is applied, selection of the phase measurement for the first interpolation depends on the interpolation error distribution within the sampling points. The interpolation error depends on the function type and interpolation method. To analyze the interpolation error variation in case of the one-way phase, a simple linear curve is generated with a Gaussian noise

$$\text{Raw}: y_i = at_i + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma^2) \quad t_i = 0, 0.01, \ldots, 1000 \quad (3)$$

$$\text{Sampling points: } t_s = t_j + b_k \quad b_k = 0, 0.01, \ldots, 0.09$$
$$t_j = 0, 0.1, \ldots, 1000 \quad (4)$$

Truth measurements y_i are generated with 0.01 s interval, and then sampling data points for the interpolation are picked at every $t_j + b_k$, where b_k is a timing offset. Values at t_j are interpolated from the sampling data. The interpolated values are compared with the truth values at t_j. Ten offsets 0.01 to 0.09 s were tested. For Monte Carlo simulations, each case runs with 100 noise realizations by changing the random number seeds. The standard deviation (STD) of the Gaussian noise varies from 0.01 to 10. Three interpolation methods, Lagrange interpolation of orders 2 and 3 and the spline method, are applied. The Lagrange interpolations of orders 2 and 3 have different error characteristics. The Lagrange interpolation of order 3 uses four points, and the interested point t_j is between two sampling points in each side (symmetric). The order 2 uses three points and t_j is between one sampling point and two sampling points (unsymmetric). Two selections of the sampling points, for example, two points before t_j or one point before t_j, are available for order 2. Depending on how t_j is close to the sampling point, t_j might be located in either the left or right end of the sampling point. For this reason, the interpolation error level with the order 2 is not well predictable in contrast to order 3. In this research, the sampling interval for order 2 is selected such that t_j is closest to the middle sampling point. By using this algorithm, the interpolation error shows certain level of consistency as order 3.

The interpolation error variation with the timing offset is shown in Fig. 3. The interpolation error is minimized when the offset is half of the sampling interval (the midpoint of sampling data points). It conflicts with the myth that smaller offset (closer to the sampling points) reduces the interpolation error. This trend is especially true for noisy data because the midpoint is closer to the average of the raw data and the noise tends to be averaged. To analyze the effect of the noise on the interpolation error magnitude, three different noise levels are applied for the Lagrange interpolation of order 3. For placing the results onto one plot, the results with $\sigma = 0.01$ and 1 are magnified by 1000 and 10 times, respectively. The interpolation error is proportional to the noise level. The Lagrange interpolation of order 2 also has the minimum error at the midpoint, but the error reduction is sharper than order 3. Other interpolation methods, for example, spline, were tested as well, but the overall results are the same.

To test the proposed interpolation method, simulated GRACE phase measurements were interpolated. The two interpolation methods were tested in computing the dual one-way phase (range)

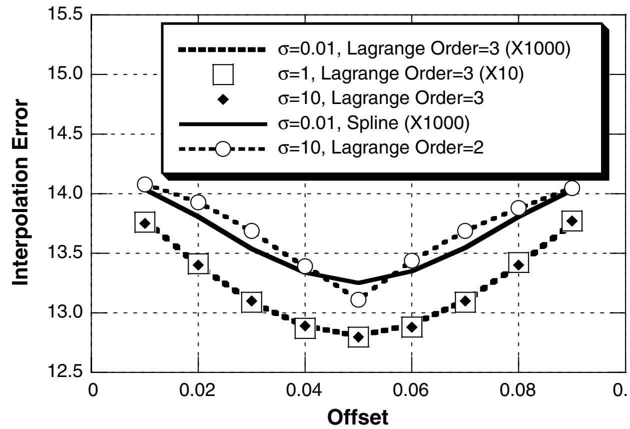
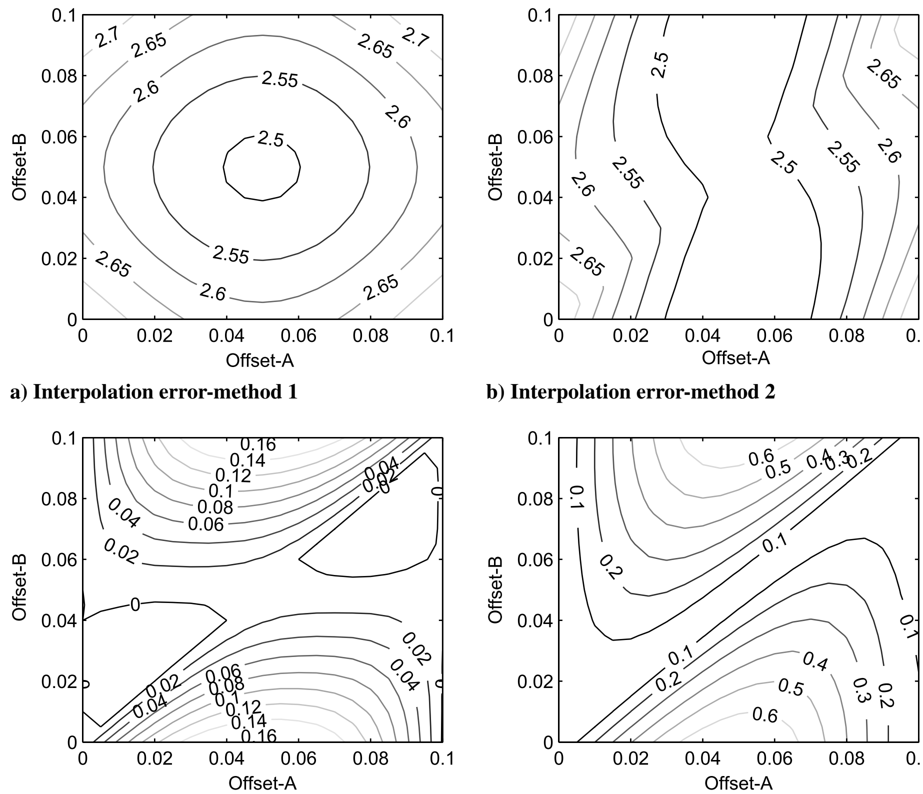


Fig. 3 Interpolation error variation with timing offset, linear curve.

measurements. One-way phase measurements were generated for the two GRACE satellites with timing offsets. The sampling interval is 0.1 s and the offset varies from 0.0 to 0.1 s with the increment of 0.005 s. The interpolated dual one-way phase is compared with the truth phase, which does not contain the timing offsets. Different offsets are applied for the two satellites, and a total of 441 offset combinations (21 by 21) are tested. Each combination was run 10 times by changing the random numbers for the phase and other noises. The results of each combination are the mean of ten realizations. The two measurements are classified as phases *A* and *B*. In case of method 2, phase *B* is interpolated first toward phase *A*'s measurement time as Fig. 2.

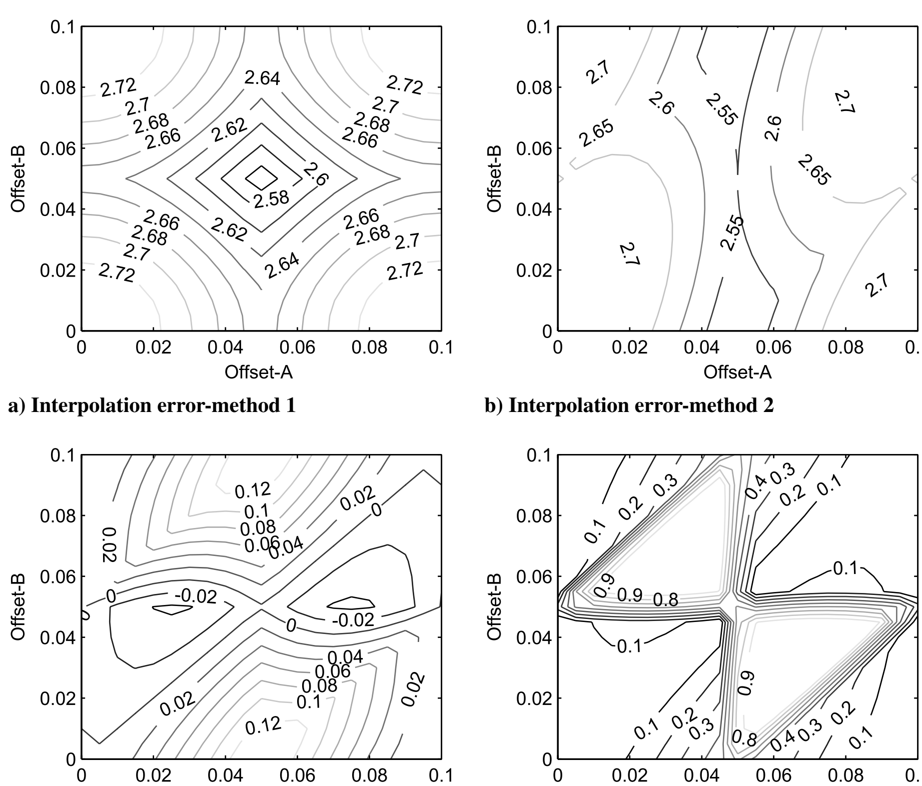
The Lagrange interpolation of order 2 or 3 is used. To minimize the numerical error due to limited number of significant digits, 16 byte real variables were used. The Jet Propulsion Laboratory uses a relative time-tag approach in order to reduce numerical error [9]. For example, the reference time of 192,110,100 s plus the timing offset of 1.0E–10 s causes loss of significant digits when using 8 byte real variables, which supports 15 digits only. Instead of the absolute time, the time differences from the beginning of the sampling point intervals, 0.2, 0.1 s, etc., are used. The same kind of algorithm was used in this research. Therefore, numerical error due to a limited number of digits can be avoided.

Figure 4 shows the STD distribution of the interpolation error by using the Lagrange interpolation of order 3. Figure 4a shows the interpolation error using method 1 for two phases' timing offset combination. The STD is computed instead of rms because the GRACE measurement is a biased range and the bias magnitude is not important. However, most of the errors have a near-zero mean value, and the STD is close to the rms. The interpolation error is around 2.5 μm, and it has a minimum when both offsets are close to the midpoint of 0.05 s. Figure 4b shows the interpolation error STD using method 2. The interpolation error has a minimum when offset *A* is close to 0.05 s. In other words, when the offset of the combined phase (offset *A*) is close to the midpoint, the second interpolation error is decreased. These results support the previous results in Fig. 3. Figure 4c shows the difference between the interpolation error STD of the two methods. The STD of method 2 is smaller than that of method 1 in all the offset combinations. It coincides with the fact that the one-way phase contains more noise than the dual one-way phase, and it is better to interpolate the dual one-way phase instead of the one-way phases to reduce the interpolation error. The interpolation error reduction by method 2 is up to 0.2 μm, approximately 10% of the interpolation error. Figure 4d shows the STD of the dual one-way range difference between the two methods. When we use real flight data, the true value is not available and only this kind of comparison is possible. The range difference has a maximum of 0.7 μm when the offset *A* is close to 0.05 s. Selection of a phase for which timing offset is closer to the midpoint as the second interpolation maximizes the difference from method 1 and minimizes the interpolation error.



a) Interpolation error-method 1
b) Interpolation error-method 2
c) Interpolation error difference (1-2)
d) Range difference (1-2)

Fig. 4 Simulated GRACE phase interpolation errors (order = 3, unit = μm).



a) Interpolation error-method 1
b) Interpolation error-method 2
c) Interpolation error difference (1-2)
d) Range difference (1-2)

Fig. 5 Simulated GRACE phase interpolation errors (order = 2, unit = μm).

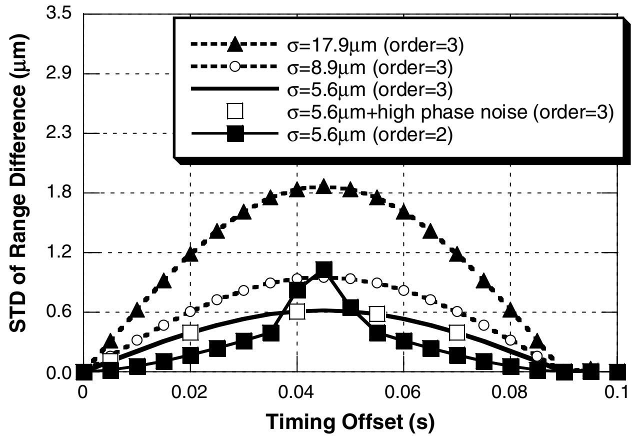


Fig. 6 Range difference between the two interpolation methods (simulated data, offset $B = 0.09$ s).

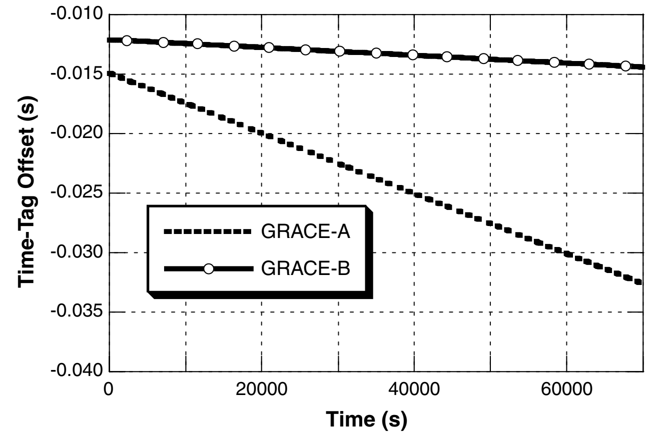


Fig. 7 GRACE timing offsets (28 Nov. 2003).

Figure 5 shows the STD distribution of the interpolation error by using the Lagrange interpolation of order 2. The maximum range error reduction (Fig. 5c) is 0.12 μm and slightly smaller than that of the order 3 results in Fig. 4. Overall distribution is similar to the order 3, for example, method 2 yields a minimum error when offset A is close to the midpoint (Figure 5b). However, the range difference in Fig. 5d shows a different pattern with a larger difference, up to 1.2 μm. The difference is maximized when one offset is higher than 0.05 s and the other is lower than 0.05 s.

More analysis was performed with different noise levels. Offset B is fixed as 0.09 s and offset A is varying from 0 to 0.09 s. Figure 6 shows the STD of the range difference between methods 1 and 2 with different system noise levels. White noise is simulated for the system noise that is caused by each receiver's hardware. The system noise level of $\sigma = 5.6$ μm corresponds to the GRACE phase signal-to-noise ratio of 69 dB for 0.1 s sampling time, which was used for the GRACE baseline performance simulations [7,8]. As the system noise level is increased, the range difference is increased. A new phase noise was generated with a higher noise level at the high frequency, above 1 Hz. However, increase of the phase noise does not change the results. It is because the phase noises of the two satellites are highly correlated and most of its high noise effect is canceled during the dual one-way combination process. The range difference with the Lagrange interpolation of order 2 is smaller than that of the order 3, although it shows a higher peak around 0.045 s. Other interpolation methods, for example, spline, were tested, but the overall results were not changed.

One limitation of this simulation analysis is that the simulated data contain some interpolation error as well. Applying the orbit and time-of-flight information involves the interpolation process. Instead of the simulated phase measurements, two time series as Eq. (3) were tested to evaluate method 2 without the interpolation error in the raw data. The results showed the same trend as the simulated phases and support the better performance of method 2.

IV. GRACE Flight Data Analysis

The two interpolation methods were applied for the GRACE flight data. Because the GRACE DOWR phase measurements are not available to the public, selected data sets were obtained from the University Texas at Austin by a special request.

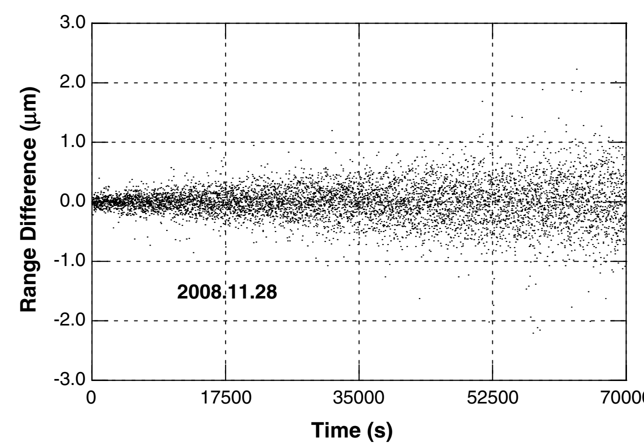


Fig. 8 Range difference between methods 1 and 2 (order = 3, 28 Nov. 2003).

Figure 7 shows the timing offset of the two GRACE satellites on 28 November 2003. GRACE A's oscillator has a larger frequency offset than GRACE B's, and it has a larger linear trend. In case of the GRACE, the onboard clock runs faster than a reference GPS time, and the timing offsets have negative values. With 0.1 s interpolation sampling time, the negative values can be mapped to positive values, for example, -0.03 s becomes 0.07 s ($=0.1 - 0.03$ s). Use of this positive value provides a better consistency with the simulation results where timing offsets were set to be positive values. With the positive value, GRACE A's offset varies from 0.085 s ($=0.1-0.015$ s) to 0.067 s ($=0.1 - 0.033$ s) whereas GRACE B's offset remains around 0.088 s on this date.

Figure 8 shows the range difference between the two interpolation methods with the Lagrange interpolation order 3. Because true (uninterpolated) values are not available, analysis of this difference is the only way to test the proposed method. For method 2, satellite B's phase is interpolated first, because offset A is closer to the midpoint, 0.05 s, than offset B. The opposite sequence, phase A's interpolation first, is tested as well, but the range difference is as small as Fig. 4d,

Table 1 Range difference STD between methods 1 and 2 for selected dates

Date	Timing offset, s		Range difference, μm, order 3		Range difference, μm, order 2	
	GRACE A	GRACE B	Flight	Simulation	Flight	Simulation
27 Nov. 2003	0.015	0.090	0.46	0.30	0.35	0.10
10 Oct. 2004	0.025	0.091	0.53	0.46	0.33	0.22
20 Oct. 2004	0.096	0.000	0.08	0.07	0.01	0.01
2 Aug. 2005	0.076	0.090	0.33	0.28	0.11	0.09
2 Feb. 2006	0.069	0.094	0.66	0.47	0.29	0.21

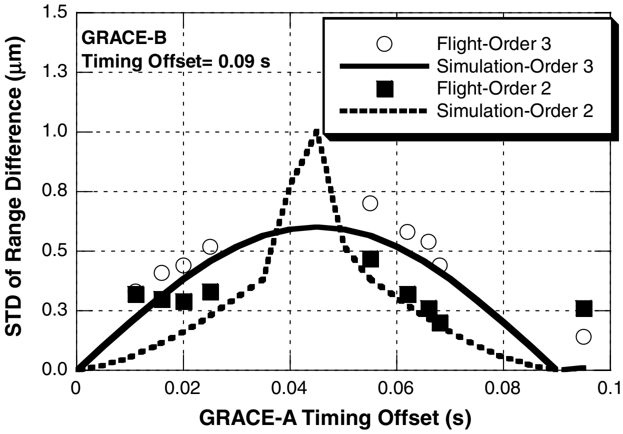


Fig. 9 Range difference between methods 1 and 2 for selected dates (offset-$B = 0.09$ s).

and the results are not presented. The range difference becomes larger as the GRACE A's timing offset increases (closer to 0.05 s). The range difference STD between 67,000 and 70,000 s is 0.50 μm and close to the simulation results in Fig. 4d. The range difference with the Lagrange interpolation of order 2, not shown in the figure, has a lower value of 0.23 μm during the same 3000 s interval, and it is also close to the simulation results in Fig. 5d.

Other dates' phase measurements were tested as well. Table 1 shows the range difference STD from the flight and simulation data for selected dates. The Lagrange interpolations of orders 2 and 3 are used. During each day, a 3000 s interval was selected for the analysis. Because the timing offset increases with time, a small time range is selected for assuming a constant offset. The variation of the timing offset during the 3000 s is less than 0.001 s. For method 2, satellite-B's phase is interpolated first. Although the STD shows a certain level of difference, the STD variation trend is close to the simulation results in an overall sense. The difference is minimized when the two offsets are close.

For more comparison, selected dates when GRACE B's offset is 0.09 s were analyzed. Figure 9 shows the range difference variation with GRACE A's timing offset. Because of the limited amount of GRACE KBR phase data available, only 10 days of flight data between October and November 2004 were analyzed. In the case of the order 3, the flight data are slightly higher than the simulation data, but the overall trend is the same. In the case of order 2, the flight data well agree with the simulation data near the midpoint, but the deviation increases as the offset becomes 0 or 0.1 s. Although it is better to compare the flight data at the midpoint (0.05 s), none of the data were available with that offset combination (offset $A = 0.05$ s, offset $B = 0.09$ s). This result validates the proposed interpolation method. If we assume the GRACE interpolation error level is close to the simulation results in Figs. 4a and 4b, approximately 2.5 μm, the interpolation error is smaller than the GRACE design noise, approximately 10 μm. Therefore, this analysis partly evaluates the GRACE interpolation error level, which cannot be directly evaluated from the flight data.

V. Conclusions

The DOWR systems require the measurement timing synchronization of two one-way phase measurements. The interpolation of the two measurements toward a common epoch resolves this requirement but causes interpolation error. A new interpolation method is proposed to reduce the interpolation error, which modifies the interpolation sequence. Instead of interpolating two measurements simultaneously, one measurement is interpolated first toward the other measurement time and then combined. The combined measurement is interpolated toward a nominal time. DOWR measurement simulations based on the GRACE DOWR specifications are performed to analyze the performance of the new method. The simulation results demonstrate the advantage of the new method over a conventional interpolation method implemented in current GRACE data processing. Experiments with the GRACE flight data are performed as well, and the results are close to the simulation results.

The interpolation error reduction by this method is at the submicrometer level, and it is not very helpful for improving current GRACE range accuracy. However, the new method is useful with other configurations, for example, higher phase noise, and this method may benefit future intersatellite missions with the dual one-way ranging systems. In addition to the interpolation error reduction, this method partly evaluates the interpolation error level in the real GRACE data processing.

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